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Conformal Hamiltonian Mechanical Equations on Contact 9- Manifolds

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Abstract

In this study, we concluded the Hamiltonian equations on $(\mathcal{M}^9, \lambda, X_H)$, being a model. Finally introduce, some geometrical and physical results on the related mechanic systems have been discussed. n classical mechanics we can describe the state of a system by specifying its Lagrangian as a function of the coordinates and their time rates of change. Hamilton realized that Lagrange's equations of motion were equivalent to a variational principle.

Keywords: Differential geometry, Contact 9- Manifolds, Hamiltonian Dynamics

1 Introduction[1]

1.1 Historical Background

Hamiltonian Mechanics was introduced in 1833 by the Irish mathematician William Rowan Hamilton. As a man inspired with knowledge at a very young age Hamilton was able to accomplish many tasks in only the 60 years he lived. While there, he studied optics, classical mechanics, dynamic methods, quaternion's, and many other topics. Hamilton remained active in the mathematical society until his death in 1865 from a severe case of gout, (1). Modern Differential Geometry is a suitable frame for studying Hamiltonian formalisms of Classical Mechanics. To show this, it is possible to find many articles and books in the relevant fields. It is well-known that the dynamics of Hamiltonian systems is characterized by a convenient vector field X defined on the tangent and cotangent bundles which are phase-spaces of velocities and momentum of a given configuration manifold. If $\mathcal{M}$ is an m-dimensional configuration manifold and $H : T\mathcal{M} \to R$ is a regular Lagrangian function, then there is a unique vector field X on $T\mathcal{M}$ such that

$$i_X\omega = dH \qquad (1)$$

where ω is the Symplectic form and H stands for Hamiltonian function. The paths of the so called Hamiltonian vector field X_H are the solutions of the Hamiltonian equations. The triple, either $(T^*\mathcal{M}, \omega, X_H)$ or $(T^*\mathcal{M}, \omega, H)$, is called Hamiltonian system on the cotangent bundle $T^*\mathcal{M}$ fixed with Symplectic form ω (2)

2 Complex on Contact 9-manifolds

2.1 Definition (3)

The contact 9-manifold (M^9, α), given a horizontal 2-form ω.

2.2 Theorem

A conformal manifold is a differentiable manifold equipped with an equivalence class of Riemann metric tensors, in which twof_1 metricsf_2 and are equivalent if and only if

$$f_2 = B^2 f_1 \qquad (1)$$

where is a smooth positive function. $B > 0$

2.3 Theorem 1

A conformal transformation is a change of coordinates such that the metric changes by $\sigma^a \to \sigma^b$

$$f_{ab}(\sigma) \to B^2(\sigma) f_{ab}(\sigma)$$

2.4 Proposition

Suppose that $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, \theta\}$, be a real coordinate system on $(\mathcal{M}, J)$. Then we denote by

$$\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}, \frac{\partial}{\partial x_7}, \frac{\partial}{\partial x_8}, \theta\right\}$$

$$\{dx_1, dx_2, dx_3, dx_4, dx_5, dx_6, dx_7, dx_8, \theta\}$$

2.5 Proposition

The following expressions are given. The dual form J^* of the above J is as follows:

$$J^*(dx_1) = B^2\cos\theta dx_5 + B^2\sin\theta dx_6 \quad , \quad J^*(dx_2) = -B^2\cos\theta dx_6 + B^2\sin\theta dx_5$$

$$J^*(dx_3) = B^2\cos\theta dx_7 + B^2\sin\theta dx_8 \quad , \quad J^*(dx_4) = -B^2\cos\theta dx_8 + \sin\theta dx_7$$

$$J^*(dx_5) = -B^{-2}\cos\theta dx_7 - B^{-2}\sin\theta dx_8 \ , \quad J^*(dx_6) = B^{-2}\cos\theta dx_8 - B^{-2}\sin\theta dx_7$$

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$$J^*(dx_7) = -B^{-2}\cos\theta dx_3 - B^{-2}\sin\theta dx_4 \ , \ J^*(dx_8) = B^{-2}\cos\theta dx_4 - B^{-2}\sin\theta dx_3 \quad (2)$$

2.6 Definition

A Hamiltonian system is a triple $(\mathcal{M};\xi)$, where $(\omega;L)$ is a Symplectic manifold and $H \in C^\infty(\mathcal{M})$ is a function, called the Hamiltonian function.

2.7 Theorem

Let $\mathcal{M}$ be m-real dimensional configuration manifold .A tensor field J on $T^*\mathcal{M}$ is called an almost complex structure on $T^*\mathcal{M}$ if at every point p of $T^*\mathcal{M}$, J is endomorphism of the tangent space $T_p{}^*(\mathcal{M})$ such that $J^2 = -1$ are complex is

$$J^{*2}(dx_i) = -dx_i \quad , \quad i = 1,2,3,4,5,6,7,8$$

J^{*2} is called almost complex manifold

3 Hamiltonian Dynamical Systems

3.1 Definition

A Hamiltonian function for a Hamiltonian vector field X on $\mathcal{M}$ is a smooth function $H : \mathcal{M} \to R$ such that $i_{X_H}\omega = dH$.

3.2 Definition

A Hamiltonian system is a triple $(M;\omega;H)$, where $(\omega;H)$ is a Symplectic manifold and $H \in C^\infty(M)$ is a function, called the Hamiltonian function. Suppose that an almost real structure, a Liouville form and a1-form on $T^*\mathcal{M}$ are shown by Φ^*, λ and ω, respectively. Then we have:

$$\omega = \frac{1}{2}(x_1dx_1 + x_2dx_2 + x_3dx_3 + x_4dx_4 + x_5dx_5 + x_6dx_6 + x_7dx_7 + x_8dx_8)$$

and

$$\lambda = \frac{1}{2}\big(x_1J^*(dx_1) + x_2J^*(dx_2) + x_3J^*(dx_3) + x_4J^*(dx_4) + x_5J^*(dx_5) + x_6J^*(dx_6) + x_7J^*(dx_7) + x_8J^*(dx_8)\big) \quad (3)$$

We substitute equation (2) in equation (3) we get:

$$\begin{aligned}\lambda = \Phi^*(\omega) = \frac{1}{2}[&x_1(B^2\cos\theta dx_5 + B^2\sin\theta dx_6)\\ &+ x_2(-B^2\cos\theta dx_6 + B^2\sin\theta dx_5)\\ &+ x_3(B^2\cos\theta dx_7 + B^2\sin\theta dx_8)\\ &+ x_4(-B^2\cos\theta dx_8 + B^2\sin\theta dx_7)\\ &+ x_5(-B^{-2}\cos\theta dx_7 - B^{-2}\sin\theta dx_8)\\ &+ x_6(B^{-2}\cos\theta dx_8 - B^{-2}\sin\theta dx_7)\\ &+ x_7(-B^{-2}\cos\theta dx_3 - B^{-2}\sin\theta dx_4)\\ &+ x_8(B^{-2}\cos\theta dx_4 - B^{-2}\sin\theta dx_3)]\end{aligned}$$

differential of λ

$$\begin{aligned}\phi = -d\lambda = -d\Big(\frac{1}{2}[&x_1(B^2\cos\theta dx_5 + B^2\sin\theta dx_6)\\ &+ x_2(-B^2\cos\theta dx_6 + B^2\sin\theta dx_5)\\ &+ x_3(B^2\cos\theta dx_7 + B^2\sin\theta dx_8)\\ &+ x_4(-B^2\cos\theta dx_8 + B^2\sin\theta dx_7)\\ &+ x_5(-B^{-2}\cos\theta dx_7 - B^{-2}\sin\theta dx_8)\\ &+ x_6(B^{-2}\cos\theta dx_8 - B^{-2}\sin\theta dx_7)\\ &+ x_7(-B^{-2}\cos\theta dx_3 - B^{-2}\sin\theta dx_4)\\ &+ x_8(B^{-2}\cos\theta dx_4 - B^{-2}\sin\theta dx_3)]\Big)\end{aligned}$$

$$\begin{aligned}= \frac{1}{2}\Big(\Big[&\frac{dx_1}{dx_1}(B^2\cos\theta dx_1\wedge dx_5 + B^2\sin\theta dx_1\wedge dx_6)\\ &+ \frac{dx_2}{dx_2}(-B^2\cos\theta dx_2\wedge dx_6 + B^2\sin\theta dx_2\wedge dx_5)\\ &+ \frac{dx_3}{dx_3}(B^2\cos\theta dx_3\wedge dx_7 + B^2\sin\theta dx_3\wedge dx_8)\\ &+ \frac{dx_4}{dx_4}(-B^2\cos\theta dx_4\wedge dx_8 + B^2\sin\theta dx_4\wedge dx_7)\\ &+ \frac{dx_5}{dx_5}(-B^{-2}\cos\theta dx_5\wedge dx_7 - B^{-2}\sin\theta dx_5\wedge dx_8)\\ &+ \frac{dx_6}{dx_6}(B^{-2}\cos\theta dx_6\wedge dx_8 - B^{-2}\sin\theta dx_6\wedge dx_7)\\ &+ \frac{dx_7}{dx_7}(-B^{-2}\cos\theta dx_7\wedge dx_3 - B^{-2}\sin\theta dx_7\wedge dx_4)\\ &+ \frac{dx_8}{dx_8}(B^{-2}\cos\theta dx_8\wedge dx_4 - B^{-2}\sin\theta dx_8\wedge dx_3)\Big]\Big)\end{aligned}$$

$$\begin{aligned}\phi = \frac{1}{2}([&(B^2\cos\theta dx_1\wedge dx_5 + B^2\sin\theta dx_1\wedge dx_6)\\ &+ (-B^2\cos\theta dx_2\wedge dx_6 + B^2\sin\theta dx_2\wedge dx_5)\\ &+ (B^2\cos\theta dx_3\wedge dx_7 + B^2\sin\theta dx_3\wedge dx_8)\\ &+ (-B^2\cos\theta dx_4\wedge dx_8 + B^2\sin\theta dx_4\wedge dx_7)\\ &+ (-B^{-2}\cos\theta dx_5\wedge dx_7 - B^{-2}\sin\theta dx_5\wedge dx_8)\\ &+ (B^{-2}\cos\theta dx_6\wedge dx_8 - B^{-2}\sin\theta dx_6\wedge dx_7)\\ &+ (-B^{-2}\cos\theta dx_7\wedge dx_3 - B^{-2}\sin\theta dx_7\wedge dx_4)\\ &+ (B^{-2}\cos\theta dx_8\wedge dx_4 - B^{-2}\sin\theta dx_8\wedge dx_3)]) \quad (4)\end{aligned}$$

It is known that if ϕ is a closed 2- form on $T^*\mathcal{M}$, then ϕ_H is also a symplectic structure on $T^*\mathcal{M}$. If Hamiltonian vector field X_H associated with Hamiltonian energy H is given by

$$X_H = X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8} \quad (6)$$

Calculates a value X_H and ϕ

$$\begin{aligned}\phi(X_H) = \frac{1}{2}([&(B^2\cos\theta dx_1\wedge dx_5 + B^2\sin\theta dx_1\wedge dx_6)\\ &+ (-B^2\cos\theta dx_2\wedge dx_6 + B^2\sin\theta dx_2\wedge dx_5)\\ &+ (B^2\cos\theta dx_3\wedge dx_7 + B^2\sin\theta dx_3\wedge dx_8)\\ &+ (-B^2\cos\theta dx_4\wedge dx_8 + B^2\sin\theta dx_4\wedge dx_7)\\ &+ (-B^{-2}\cos\theta dx_5\wedge dx_7 - B^{-2}\sin\theta dx_5\wedge dx_8)\\ &+ (B^{-2}\cos\theta dx_6\wedge dx_8 - B^{-2}\sin\theta dx_6\wedge dx_7)\\ &+ (-B^{-2}\cos\theta dx_7\wedge dx_3 - B^{-2}\sin\theta dx_7\wedge dx_4)\\ &+ (B^{-2}\cos\theta dx_8\wedge dx_4 - B^{-2}\sin\theta dx_8\wedge dx_3)])\Big(X^1\frac{\partial}{\partial x_1}\\ &+ X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7}\\ &+ X^8\frac{\partial}{\partial x_8}\Big)\end{aligned}$$

And

$$i_{X_H}\phi = \phi(X_H)$$

$$
\begin{aligned}
&(\mathrm{B}^2\cos\theta dx_1\wedge dx_5 + \mathrm{B}^2\sin\theta dx_1\wedge dx_6)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+ (-\mathrm{B}^2\cos\theta dx_2\wedge dx_6 + \mathrm{B}^2\sin\theta dx_2\wedge dx_5)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)(\mathrm{B}^2\cos\theta dx_3\wedge dx_7\\
&+ \mathrm{B}^2\sin\theta dx_3\wedge dx_8)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+ (-\mathrm{B}^2\cos\theta dx_4\wedge dx_8 + \mathrm{B}^2\sin\theta dx_4\wedge dx_7)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+ (-\mathrm{B}^{-2}\cos\theta dx_5\wedge dx_1 - \mathrm{B}^{-2}\sin\theta dx_5\wedge dx_2)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+ (\mathrm{B}^{-2}\cos\theta dx_6\wedge dx_2 - \mathrm{B}^{-2}\sin\theta dx_6\wedge dx_1)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+ (-\mathrm{B}^{-2}\cos\theta dx_7\wedge dx_3 - \mathrm{B}^{-2}\sin\theta dx_7\wedge dx_4)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+ (\mathrm{B}^{-2}\cos\theta dx_8\wedge dx_4 - \mathrm{B}^{-2}\sin\theta dx_8\wedge dx_3)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)
\end{aligned}
$$

or

$$
\begin{aligned}
&(dx_1\mathrm{B}^2\cos\theta dx_5 + dx_1\mathrm{B}^2\sin\theta dx_6 - dx_5\mathrm{B}^2\cos\theta dx_1 - dx_6\mathrm{B}^2\sin\theta dx_1)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+(dx_3\mathrm{B}^2\cos\theta dx_7 + dx_3\mathrm{B}^2\sin\theta dx_8 - dx_7\mathrm{B}^2\cos\theta dx_8 - dx_8\mathrm{B}^2\sin\theta dx_3)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+(-dx_4\mathrm{B}^2\cos\theta dx_8 + dx_4\mathrm{B}^2\sin\theta dx_7 + dx_8\mathrm{B}^2\cos\theta dx_4 + dx_7\mathrm{B}^2\sin\theta dx_4)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+(-dx_5\mathrm{B}^2\cos\theta dx_1 - dx_5\mathrm{B}^2\sin\theta dx_2 + dx_1\mathrm{B}^2\cos\theta dx_5 + dx_2\mathrm{B}^2\sin\theta dx_5)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+(dx_6\mathrm{B}^2\cos\theta dx_1 - dx_6\mathrm{B}^2\sin\theta dx_1 - dx_2\mathrm{B}^2\cos\theta dx_6 - dx_1\mathrm{B}^2\sin\theta dx_6)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+(-dx_7\mathrm{B}^2\cos\theta dx_3 - dx_7\mathrm{B}^2\sin\theta dx_4 + dx_3\mathrm{B}^2\cos\theta dx_7 + dx_4\mathrm{B}^2\sin\theta dx_7)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)\\
&+(dx_8\mathrm{B}^2\cos\theta dx_4 - dx_8\mathrm{B}^2\sin\theta dx_3 - dx_4\mathrm{B}^2\cos\theta dx_8 + dx_3\mathrm{B}^2\sin\theta dx_8)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right)
\end{aligned}
$$

and so we get

$\frac{1}{2}$ ($X^1B^2\cos\theta dx_5 + X^1 B^2\sin\theta dx_6 + X^1B^2\cos\theta dx_5 + X^1 B^2\sin\theta dx_6 - X^2B^2\cos\theta dx_6 - X^2B^2\sin\theta dx_5 - X^2B^2\cos\theta dx_6 - X^2 B^2\sin\theta dx_5 + X^3 B^2\cos\theta dx_7 + X^3B^2\sin\theta dx_8 + X^3B^2\cos\theta dx_8 + X^3B^2\sin\theta dx_8 - X^4B^2\cos\theta dx_8 - X^4B^2\sin\theta dx_7 - X^4 B^2\cos\theta dx_8 - X^4 B^2\sin\theta dx_7$
$- X^5B^{-2}\cos\theta dx_1 + X^5B^{-2}\sin\theta dx_2 - X^5 B^{-2}\cos\theta dx_1 + X^5 B^{-2}\sin\theta dx_2 + X^6B^{-2}\cos\theta dx_1 - X^6B^{-2}\sin\theta dx_2 + X^6 B^{-2}\cos\theta dx_1 - X^6 B^{-2}\sin\theta dx_2$
$- X^7B^{-2}\cos\theta dx_3 - X^7B^{-2}\sin\theta dx_4 - X^7 B^{-2}\cos\theta dx_3 - X^7 B^{-2}\sin\theta dx_4 + X^8B^{-2}\cos\theta dx_4 - X^8B^{-2}\sin\theta dx_3 - X^8 B^{-2}\cos\theta dx_3 + X^8 B^{-2}\sin\theta dx_4$)

or

$$X^1B^2\cos\theta dx_5 + X^1 B^2\sin\theta dx_6 - X^2B^2\cos\theta dx_6 - X^2B^2\sin\theta dx_5 - X^3 B^2V\cos\theta dx_7 + X^3B^2\sin\theta dx_8 - X^4B^2\cos\theta dx_8 - X^4B^2\sin\theta dx_7 - X^5B^{-2}\cos\theta dx_1 + X^5B^{-2}\sin\theta dx_2 + X^6B^{-2}\cos\theta dx_1 - X^6B^{-2}\sin\theta dx_2 + X^7B^{-2}\cos\theta dx_3 - X^7B^{-2}\sin\theta dx_4 + X^8B^{-2}\cos\theta dx_4 - X^8B^{-2}\sin\theta dx_3 \quad (8)$$

Moreover, the differential of Hamiltonian energy is written as follows:

$$dH = \frac{\partial H}{\partial x_1}dx_1 + \frac{\partial H}{\partial x_2}dx_2 + \frac{\partial H}{\partial x_3}dx_3 + \frac{\partial H}{\partial x_4}dx_4 + \frac{\partial H}{\partial x_5}dx_5 + \frac{\partial H}{\partial x_6}dx_6 + \frac{\partial H}{\partial x_7}dx_7 + \frac{\partial H}{\partial x_8}dx_8 \quad (9)$$

By means of Eq. (1), using Eqs. (7) and (8), the Hamiltonian vector field is calculated to be

$$X^1\cos\theta dx_5 + X^1 \sin\theta dx_6 - X^2\cos\theta dx_6 - X^2\sin\theta dx_5 - X^3 \cos\theta dx_7 + X^3\sin\theta dx_8 - X^4\cos\theta dx_8 - X^4\sin\theta dx_7 - X^5\cos\theta dx_1 + X^5\sin\theta dx_2 + X^6\cos\theta dx_1 - X^6\sin\theta dx_2 + X^7\cos\theta dx_3 - X^7\sin\theta dx_4 + X^8\cos\theta dx_4 - X^8\sin\theta dx_3 = \frac{\partial H}{\partial x_1}dx_1 + \frac{\partial H}{\partial x_2}dx_2 + \frac{\partial H}{\partial x_3}dx_3 + \frac{\partial H}{\partial x_4}dx_4 + \frac{\partial H}{\partial x_5}dx_5 + \frac{\partial H}{\partial x_6}dx_6 + \frac{\partial H}{\partial x_7}dx_7 + \frac{\partial H}{\partial x_8}dx_8$$

or

$$X^1\cos\theta dx_5 + X^1 \sin\theta dx_6 - X^2\cos\theta dx_6 - X^2\sin\theta dx_5 + X^3 \cos\theta dx_7 + X^3\sin\theta dx_8 - X^4 \cos\theta dx_8 - X^4 \sin\theta dx_7 - X^5 \cos\theta dx_2 + X^5 \sin\theta dx_1 X^6\cos\theta dx_1 - X^6\sin\theta dx_2 + X^6 \cos\theta dx_2 - X^7\cos\theta dx_3 - X^7\sin\theta dx_4 - X^8\cos\theta dx_4 - X^8\sin\theta dx_3 = \frac{\partial H}{\partial x_1}dx_1 + \frac{\partial H}{\partial x_2}dx_2 + \frac{\partial H}{\partial x_3}dx_3 + \frac{\partial H}{\partial x_4}dx_4 + \frac{\partial H}{\partial x_5}dx_5 + \frac{\partial H}{\partial x_6}dx_6 + \frac{\partial H}{\partial x_7}dx_7 + \frac{\partial H}{\partial x_8}dx_8$$

By comparing the two sides we get the following:

$$X^5B^{-2}\cos\theta dx_1 + X^6 B^{-2}\sin\theta dx_1 = B^{-2}(X^5\cos\theta + X^6 \sin\theta)dx_1 = \frac{\partial H}{\partial x_1}dx_1 \Rightarrow B^{-2}(X^5\cos\theta + X^6 \sin\theta) = \frac{\partial H}{\partial x_1}$$

$$X^5B^{-2}\sin\theta dx_2 - X^6 B^{-2}\cos\theta dx_2 = B^{-2}(X^5\sin\theta - X^6 \cos\theta)dx_2 = \frac{\partial H}{\partial x_2}dx_2 \Rightarrow B^{-2}(X^5\sin\theta - X^6 \cos\theta) = \frac{\partial H}{\partial x_2}$$

$$-X^7B^{-2}\cos\theta dx_3 + X^8 B^{-2}\sin\theta dx_3 = B^{-2}(-X^7\cos\theta + X^8 \sin\theta)dx_3 = \frac{\partial H}{\partial x_3}dx_3 \Rightarrow B^{-2}(-X^7\cos\theta + X^8 \sin\theta) = \frac{\partial H}{\partial x_3}$$

$$X^7B^{-2}\sin\theta dx_4 - X^8B^{-2} \cos\theta dx_4 = B^{-2}(X^7\sin\theta - X^8 \cos\theta)dx_4 = \frac{\partial H}{\partial x_4}dx_4 \Rightarrow B^{-2}(X^7\sin\theta - X^8 \cos\theta) = \frac{\partial H}{\partial x_4}$$

$$-X^1B^2\cos\theta dx_5 - X^2B^2 \sin\theta dx_5 = B^2(-X^1\cos\theta - X^2 \sin\theta)dx_5 = \frac{\partial H}{\partial x_5}dx_5 \Rightarrow B^2(-X^1\cos\theta - X^2 \sin\theta) = \frac{\partial H}{\partial x_5}$$

$$-X^1B^2\sin\theta dx_6 + X^2B^2V \cos\theta dx_6 = B^2(-X^1\sin\theta + X^2 \cos\theta)dx_6 = \frac{\partial H}{\partial x_6}dx_6 \Rightarrow B^2(-X^1\sin\theta + X^2 \cos\theta) = \frac{\partial H}{\partial x_6}$$

$$-X^3B^2\cos\theta dx_7 - X^4 B^2\sin\theta dx_7 = B^2(-X^3\cos\theta - X^4 \sin\theta)dx_7 = \frac{\partial H}{\partial x_7}dx_7 \Rightarrow B^2(-X^3\sin\theta + X^4 \cos\theta) = \frac{\partial H}{\partial x_7}$$

$$-X^3B^2\sin\theta dx_8 + X^4 B^2\cos\theta dx_8 = B^2 (-X^3\sin\theta + X^4 \cos\theta)dx_8 = \frac{\partial H}{\partial x_7}dx_7 \Rightarrow B^2(-X^3\sin\theta + X^4 \cos\theta) = \frac{\partial H}{\partial x_8} \quad (10)$$

So we find that

$$X^1 = - B^2\cos\theta\frac{\partial H}{\partial x_5} - B^2\, sin\,\theta\frac{\partial H}{\partial x_6}$$
$$X^2 = B^2 \cos\theta\frac{\partial H}{\partial x_6} - B^2 \sin\theta\frac{\partial H}{\partial x_5}$$
$$X^3 = -B^2\cos\theta\frac{\partial H}{\partial x_7} - B^2 \sin\theta\frac{\partial H}{\partial x_8}$$
$$X^4 = B^2 \cos\theta\frac{\partial H}{\partial x_8} - B^2 \sin\theta\frac{\partial H}{\partial x_7}$$
$$X^5 = B^{-2} \cos\theta\frac{\partial H}{\partial x_1} + B^{-2} \sin\theta\frac{\partial H}{\partial x_2}$$

$$X^6 = -B^{-2}\cos\theta\frac{\partial H}{\partial x_2} + B^{-2}\sin\theta\frac{\partial H}{\partial x_1}$$
$$X^7 = B^{-2}\cos\theta\frac{\partial H}{\partial x_3} + B^{-2}\sin\theta\frac{\partial H}{\partial x_4}$$
$$X^8 = -B^{-2}\cos\theta\frac{\partial H}{\partial x_4} + B^{-2}\sin\theta\frac{\partial H}{\partial x_3} \quad (11)$$

Equation compensate (11) in equation (6) we get

$$B^2\left(-\cos\theta\frac{\partial H}{\partial x_5} - \sin\theta\frac{\partial H}{\partial x_6}\right)\frac{\partial}{\partial x_1} + B^2\left(\cos\theta\frac{\partial H}{\partial x_6} - \sin\theta\frac{\partial H}{\partial x_5}\right)\frac{\partial}{\partial x_2} + B^2\left(-\cos\theta\frac{\partial H}{\partial x_7} - \sin\theta\frac{\partial H}{\partial x_8}\right)\frac{\partial}{\partial x_3} + B^2\left(\cos\theta\frac{\partial H}{\partial x_8} - \sin\theta\frac{\partial H}{\partial x_7}\right)\frac{\partial}{\partial x_4} + B^2\left(\cos\theta\frac{\partial H}{\partial x_1} + \sin\theta\frac{\partial H}{\partial x_2}\right)\frac{\partial}{\partial x_5} + B^{-2}\left(-\cos\theta\frac{\partial H}{\partial x_2} + \sin\theta\frac{\partial H}{\partial x_1}\right)\frac{\partial}{\partial x_6} + B^{-2}\left(\cos\theta\frac{\partial H}{\partial x_3} + \sin\theta\frac{\partial H}{\partial x_4}\right)\frac{\partial}{\partial x_7} + B^{-2}\left(-\cos\theta\frac{\partial H}{\partial x_4} + \sin\theta\frac{\partial H}{\partial x_3}\right)\frac{\partial}{\partial x_8} = \frac{\partial H}{\partial x_1}dx_1 + \frac{\partial H}{\partial x_2}dx_2 + \frac{\partial H}{\partial x_3}dx_3 + \frac{\partial H}{\partial x_4}dx_4 + \frac{\partial H}{\partial x_5}dx_5 + \frac{\partial H}{\partial x_6}dx_6 + \frac{\partial H}{\partial x_7}dx_7 + \frac{\partial H}{\partial x_8}dx_8 \quad (12)$$

Suppose that a curve $\alpha: I \subset R \rightarrow T^*\mathcal{M} = R^{2n}$ is an integral curve of the Hamiltonian vector field X_H, i.e.,

$$X_H(\alpha(t)) = \frac{d\alpha(t)}{dt} \quad , \quad t \in I.$$

In the local coordinates, if it is considered to be

$$\alpha(t) = (x_1(t), x_2(t), x_2(t), x_4(t), x_5(t), x_6(t), x_7(t), x_8(t))$$

We obtain

$$\frac{d\alpha(t)}{dt} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3} + \frac{dx_4}{dt}\frac{\partial}{\partial x_4} + \frac{dx_5}{dt}\frac{\partial}{\partial x_5} + \frac{dx_6}{dt}\frac{\partial}{\partial x_6} + \frac{dx_7}{dt}\frac{\partial}{\partial x_7} + \frac{dx_8}{dt}\frac{\partial}{\partial x_8} \quad (13)$$

Taking the equation (12) = the equation (13)

$$B^2\left(-\cos\theta\frac{\partial H}{\partial x_5} - \sin\theta\frac{\partial H}{\partial x_6}\right)\frac{\partial}{\partial x_1} + B^2\left(\cos\theta\frac{\partial H}{\partial x_6} - \sin\theta\frac{\partial H}{\partial x_5}\right)\frac{\partial}{\partial x_2} + B^2\left(-\cos\theta\frac{\partial H}{\partial x_7} - \sin\theta\frac{\partial H}{\partial x_8}\right)\frac{\partial}{\partial x_3} + B^2\left(\cos\theta\frac{\partial H}{\partial x_8} - \sin\theta\frac{\partial H}{\partial x_7}\right)\frac{\partial}{\partial x_4} + B^{-2}\left(\cos\theta\frac{\partial H}{\partial x_1} + \sin\theta\frac{\partial H}{\partial x_2}\right)\frac{\partial}{\partial x_5} + B^{-2}\left(-\cos\theta\frac{\partial H}{\partial x_2} + \sin\theta\frac{\partial H}{\partial x_1}\right)\frac{\partial}{\partial x_6} + B^{-2}\left(\cos\theta\frac{\partial H}{\partial x_3} + \sin\theta\frac{\partial H}{\partial x_4}\right)\frac{\partial}{\partial x_7} + B^{-2}\left(-\cos\theta\frac{\partial H}{\partial x_4} + \sin\theta\frac{\partial H}{\partial x_3}\right)\frac{\partial}{\partial x_8} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3} + \frac{dx_4}{dt}\frac{\partial}{\partial x_4} + \frac{dx_5}{dt}\frac{\partial}{\partial x_5} + \frac{dx_6}{dt}\frac{\partial}{\partial x_6} + \frac{dx_7}{dt}\frac{\partial}{\partial x_7} + \frac{dx_8}{dt}\frac{\partial}{\partial x_8}$$

By comparing the two sides of the equation we get the

$$B^2\left(-\cos\theta\frac{\partial H}{\partial x_5} - \sin\theta\frac{\partial H}{\partial x_6}\right)\frac{\partial}{\partial x_1} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1}$$
$$-B^2\cos\theta\frac{\partial H}{\partial x_5} - B^2\sin\theta\frac{\partial H}{\partial x_6} = \frac{dx_1}{dt}$$

and

$$B^2\left(\cos\theta\frac{\partial H}{\partial x_6} - \sin\theta\frac{\partial H}{\partial x_5}\right)\frac{\partial}{\partial x_2} = \frac{dx_2}{dt}\frac{\partial}{\partial x_2}$$
$$B^2\cos\theta\frac{\partial H}{\partial x_6} - B^2\sin\theta\frac{\partial H}{\partial x_5} = \frac{dx_2}{dt}$$

and

$$B^2\left(-\cos\theta\frac{\partial H}{\partial x_7} - \sin\theta\frac{\partial H}{\partial x_8}\right)\frac{\partial}{\partial x_3} = \frac{dx_3}{dt}\frac{\partial}{\partial x_3}$$
$$-B^2\cos\theta\frac{\partial H}{\partial x_7} - B^2\sin\theta\frac{\partial H}{\partial x_8} = \frac{dx_3}{dt}$$

and

$$B^2\left(\cos\theta\frac{\partial H}{\partial x_8} - \sin\theta\frac{\partial H}{\partial x_7}\right)\frac{\partial}{\partial x_4} = \frac{dx_4}{dt}\frac{\partial}{\partial x_4}$$
$$B^2\cos\theta\frac{\partial H}{\partial x_8} - B^2\sin\theta\frac{\partial H}{\partial x_7} = \frac{dx_4}{dt}$$

and

$$B^{-2}\left(\cos\theta\frac{\partial H}{\partial x_1} + \sin\theta\frac{\partial H}{\partial x_2}\right)\frac{\partial}{\partial x_5} = \frac{dx_5}{dt}\frac{\partial}{\partial x_5}$$
$$B^{-2}\cos\theta\frac{\partial H}{\partial x_1} + B^{-2}\sin\theta\frac{\partial H}{\partial x_2} = \frac{dx_5}{dt}$$

and

$$B^{-2}\left(-\cos\theta\frac{\partial H}{\partial x_2} + \sin\theta\frac{\partial H}{\partial x_1}\right)\frac{\partial}{\partial x_6} = \frac{dx_6}{dt}\frac{\partial}{\partial x_6}$$
$$-B^{-2}\cos\theta\frac{\partial H}{\partial x_2} + B^{-2}\sin\theta\frac{\partial H}{\partial x_1} = \frac{dx_6}{dt}$$

and

$$B^{-2}\left(\cos\theta\frac{\partial H}{\partial x_3} + \sin\theta\frac{\partial H}{\partial x_4}\right)\frac{\partial}{\partial x_7} = \frac{dx_7}{dt}\frac{\partial}{\partial x_7}$$
$$B^{-2}\cos\theta\frac{\partial H}{\partial x_3} + B^{-2}\sin\theta\frac{\partial H}{\partial x_4} = \frac{dx_7}{dt}$$

and

$$B^{-2}\left(-\cos\theta\frac{\partial H}{\partial x_4} + B^2\sin\theta\frac{\partial H}{\partial x_3}\right)\frac{\partial}{\partial x_8} = \frac{dx_8}{dt}\frac{\partial}{\partial x_8}$$
$$-B^{-2}\cos\theta\frac{\partial H}{\partial x_4} + B^{-2}\sin\theta\frac{\partial H}{\partial x_3} = \frac{dx_8}{dt}$$

Thus Hamilton's equations are

$$-B^2\cos\theta\frac{\partial H}{\partial x_5} - B^2\sin\theta\frac{\partial H}{\partial x_6} = \frac{dx_1}{dt} \quad , \quad B^2\cos\theta\frac{\partial H}{\partial x_6} - B^2\sin\theta\frac{\partial H}{\partial x_5} = \frac{dx_2}{dt}$$

$$-B^2\cos\theta\frac{\partial H}{\partial x_7} - B^2\sin\theta\frac{\partial H}{\partial x_8} = \frac{dx_3}{dt} \quad , \quad B^2\cos\theta\frac{\partial H}{\partial x_8} - B^2\sin\theta\frac{\partial H}{\partial x_7} = \frac{dx_4}{dt}$$

$$B^{-2}\cos\theta\frac{\partial H}{\partial x_1} + B^2\sin\theta\frac{\partial H}{\partial x_2} = \frac{dx_5}{dt} \quad , -B^{2-}\cos\theta\frac{\partial H}{\partial x_2} + B^{-2}\sin\theta\frac{\partial H}{\partial x_1} = \frac{dx_6}{dt}$$

$$B^{-2}\cos\theta\frac{\partial H}{\partial x_3} + B^{-2}\sin\theta\frac{\partial H}{\partial x_4} = \frac{dx_7}{dt} \quad , -B^{-2}\cos\theta\frac{\partial H}{\partial x_4} + B^{-2}\sin\theta\frac{\partial H}{\partial x_3} = \frac{dx_8}{dt} \qquad (14)$$

Hence the triple $(T^*\mathcal{M} = \mathcal{M}^9, \phi_H, X_H)$ is shown to be a Hamiltonian mechanical system which are deduced by means of an almost real structure j^* and using of basis $\left\{\frac{\partial}{\partial x_i} : i = 1,2,3,4,5,6,7,8\right\}$ on the distributions $T^*\mathcal{M}$

4 Conclusions

The solutions of the Hamiltonian equations determined by (14) on the mechanical system on 9-contact manifolds $(\mathcal{M}^9, \omega, X_H)$ are the paths of vector field X_H on $\mathcal{M}^9$. To construct Hamilton's canonical equations for a mechanical system proceed as follows: 1. Choose your generalized coordinates q = (q1, . . . , qn) T and construct.

2. Define and compute the generalized momenta
3. Construct and compute the Hamiltonian function
4. Write down Hamilton's equations of motion

Ethical issue

Authors are aware of, and comply with, best practice in publication ethics specifically with regard to authorship (avoidance of guest authorship), dual submission, manipulation of figures, competing interests and compliance with policies on research ethics. Authors adhere to publication requirements that submitted work is original and has not been published elsewhere in any language.

Competing interests

The authors declare that there is no conflict of interest that would prejudice the impartiality of this scientific work.

Authors' contribution

All authors of this study have a complete contribution for data collection, data analyses and manuscript writing.

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